C2 MAY 2013 (R)

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1. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

$y = 2x + 3 + 8x^{-2} \Rightarrow y' = 2 - 16x^{-3} = 2 - \frac{16}{x^3}$

at stat points $y' = 0 \Rightarrow 2 = \frac{16}{x^3} \Rightarrow x^3 = 8$

$\therefore x = 2$

$y = 2(2) + 3 + \frac{8}{2^2} = 9$ $(2, 9)$

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2. $$y = \frac{x}{\sqrt{(1+x)}}$$

(a) Complete the table below with the value of y corresponding to $x =$ answer to 4 decimal places.

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090	0.8572	0.9037	0.9487

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\,dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

$\frac{1}{2}(0.1)\left[0.7071 + 0.9487 + 2(0.7591 + 0.8090 + 0.8572 + 0.9037)\right]$

$\underline{0.416}$

3. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(4)

$(a+b)^8 = a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3$

$\therefore \left(2-\tfrac{1}{2}x\right)^8 = 2^8 + 8(2)^7\left(-\tfrac{1}{2}x\right) + 28(2)^6\left(-\tfrac{1}{2}x\right)^2 + 56(2)^5\left(-\tfrac{1}{2}x\right)^3$

$= 256 - 512x + 448x^2 - 224x^3$

4. $\mathrm{f}(x) = ax^3 - 11x^2 + bx + 4$, where a an[...]tants.

When $\mathrm{f}(x)$ is divided by $(x - 3)$ the remainder is 55

When $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is -9

(a) Find the value of a and the value of b.

Given that $(3x + 2)$ is a factor of $\mathrm{f}(x)$,

(b) factorise $\mathrm{f}(x)$ completely.

(4)

$f(3) = 27a - 99 + 3b + 4 = 55 \Rightarrow 27a + 3b = 150$

$\Rightarrow 9a + b = 50$

$f(-1) = -a - 11 - b + 4 = -9 \Rightarrow \underline{a + b = 2}$

$8a = 48 \Rightarrow a = 6$

$b = -4$

b)

	$2x^2$	$-5x$	$+2$
$3x$	$6x^3$	$-15x^2$	$+6x$
$+2$	$4x^2$	$-10x$	$+4$

$r = 0$

$(3x+2)(2x-1)(x-2)$

5. The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$ **(4)**

(b) Hence show that $p = 5$ **(2)**

(c) Find the common ratio of this series. **(2)**

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer. **(3)**

$\frac{3p+15}{4p} = \frac{5p+20}{3p+15} \Rightarrow 9p^2+90p+225 = 20p^2+80p$

$\Rightarrow 11p^2-10p-225=0$

b) $(11p+45)(p-5)=0 \Rightarrow p=5,\ p=-\frac{45}{11}$

$\therefore \underline{p=5}$

c) $\frac{3(5)+15}{4(5)} = \frac{30}{20} = 1.5$

d) $S_{10} = \frac{20(1-1.5^{10})}{1-1.5} \simeq \underline{2267}$

6. Given that $\log_3 x = a$, find in terms of a,

(a) $\log_3 (9x)$

(b) $\log_3\left(\frac{x^5}{81}\right)$ **(3)**

giving each answer in its simplest form.

(c) Solve, for x,

$$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures. **(4)**

a) $\log_3(9x) = \log_3 9 + \log_3 x \Rightarrow 2+a$

b) $\log_3\left(\frac{x^5}{81}\right) = \log_3 x^5 - \log_3 81 = 5a-4$

c) $(2+a)+(5a-4)=3$

$\Rightarrow 6a=5 \quad \therefore a=0.8333$

$\log_3 x = \frac{5}{6} \Rightarrow x = 3^{\frac{5}{6}} = \underline{2.498}$

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7.

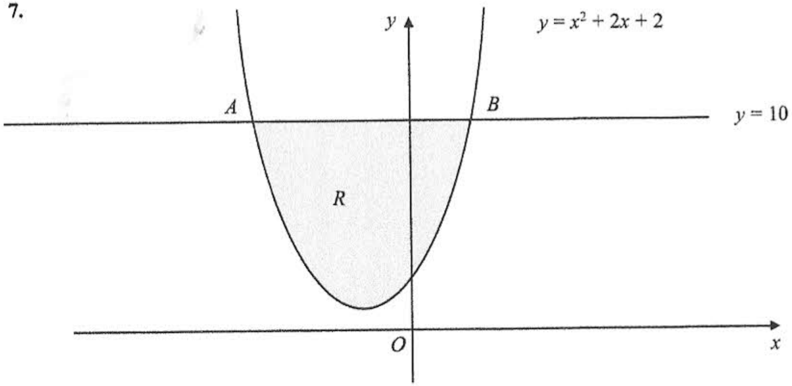

Figure 1

The line with equation $y = 10$ cuts the curve with equation $y = x^2 + 2x + 2$ at the points A and B as shown in Figure 1. The figure is not drawn to scale.

(a) Find by calculation the x-coordinate of A and the x-coordinate of B. **(2)**

The shaded region R is bounded by the line with equation $y = 10$ and the curve as shown in Figure 1.

(b) Use calculus to find the exact area of R. **(7)**

$x^2+2x+2=10 \Rightarrow x^2+2x-8=0 \Rightarrow (x+4)(x-2)=0$

$x=-4, x=2$

b) $R = {}_{10}\boxed{\quad}_{6} - \int_{-4}^{2} x^2+2x+2\,dx$

$60 - \left[\frac{1}{3}x^3+x^2+2x\right]_{-4}^{2} = 60 - \left[\left(\frac{32}{3}\right)-\left(\frac{-40}{3}\right)\right]$

$= 60 - 24 \quad = \underline{36}$

8.

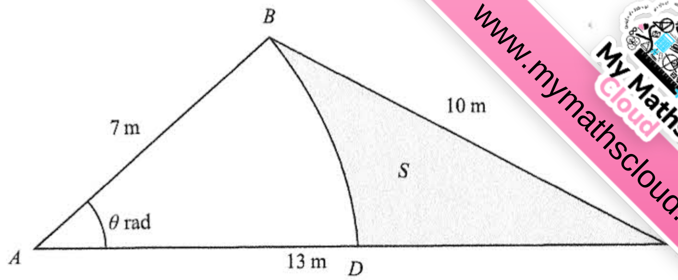

Figure 2

Figure 2 shows the design for a triangular garden ABC where $AB = 7$ m, $AC = 13$ m and $BC = 10$ m.

Given that angle $BAC = \theta$ radians,

(a) show that, to 3 decimal places, $\theta = 0.865$ **(3)**

The point D lies on AC such that BD is an arc of the circle centre A, radius 7 m.

The shaded region S is bounded by the arc BD and the lines BC and DC. The shaded region S will be sown with grass seed, to make a lawned area.

Given that 50 g of grass seed are needed for each square metre of lawn,

(b) find the amount of grass seed needed, giving your answer to the nearest 10 g. **(7)**

a) $\cos\theta = \dfrac{7^2+13^2-10^2}{2\times 7\times 13} \quad \therefore \theta = 0.865^c$

b) $S = \frac{1}{2}(7)(13)\sin 0.865 - \frac{1}{2}(7)^2(0.865)$

$S = 13.4392...$

Seed $= 671.96..$ require 670g of seed.

9. (i) Solve, for $0 \leqslant \theta < 180°$

$$\sin(2\theta - 30°) + 1 = 0.4$$

giving your answers to 1 decimal place.

(3)

(ii) Find all the values of x, in the interval $0 \leqslant x < 360°$, for which

$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0$$

giving your answers to 1 decimal place.

(7)

You must show clearly how you obtained your answers.

a) $2\theta - 30 = \sin^{-1}(-0.6) = -36.87, 216.87, 323.13$

$+30 \Rightarrow 2\theta = 246.87, 353.13$

$\theta = 123.4, 176.6$

b) $9\cos^2 x - 11\cos x + 3 - 3\cos^2 x = 0$

$6\cos^2 x - 11\cos x + 3 = 0$

$(3\cos x - 1)(2\cos x - 3) = 0$

$\cos x = \frac{1}{3}$ $\quad\quad$ $\cos x = \frac{3}{2}$

no solutions

$x = 70.5, 289.5$